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$$L = \frac{CS}{2} \text{ and } l = \frac{cs}{2}. \quad \therefore L - l = \frac{CS - cs}{2}.$$

$$\frac{S}{s} = \frac{R}{r} = \frac{C}{c}. \quad \therefore Sc = Cs.$$

$$\therefore Sc - Cs = 0. \quad CS - cs = CS - cs.$$

$$\therefore CS + Sc - Cs - cs = CS - cs. \quad \therefore (C + c)(S - s) = CS - cs.$$

$$\therefore L - l = \frac{(S - s)(C + c)}{2}. \quad \text{Q. E. D.}$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Complete the frustum to a full cone, and by cutting along an element spread out the surface into a plane, which can be done with all developable surfaces. The circular arc AB is equal to the circumference of one of the base circles of the frustum $= 2\pi R$, and arc $CD = 2\pi r$.

$AC = BD =$ slant height l of frustum.

From $OA : OC = AB : CD = R : r$, and $OA - OC = l$, we get

$$OA = \frac{Rl}{R - r}, \quad OC = \frac{rl}{R - r}.$$

$$\begin{aligned} \text{Area of } ABCD &= \frac{1}{2}AB \cdot OA - \frac{1}{2}CD \cdot OC = \frac{1}{2} \cdot 2\pi R \cdot \frac{Rl}{R - r} - \frac{1}{2} \cdot 2\pi r \cdot \frac{rl}{R - r} = \\ &= \pi l \cdot \frac{R^2 - r^2}{R - r} = \pi l(R + r). \end{aligned}$$

CALCULUS.

277. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

Find $\frac{d^2x}{ds^2}$ and $\frac{d^2y}{ds^2}$ for $y = c \sinh \frac{x}{c}$.

Solution by J. W. CLAWSON, Collegeville, Pa.

$$y = c \sinh \frac{x}{c}. \quad \therefore \frac{dy}{dx} = \cosh \frac{x}{c}. \quad \text{But } ds^2 = dx^2 + dy^2.$$

$$\therefore \left(\frac{ds}{dx} \right)^2 = 1 + \cosh^2 \frac{x}{c} \quad \text{and} \quad \left(\frac{ds}{dy} \right)^2 = 1 + \frac{1}{\cosh^2 (x/c)}.$$

$$\therefore \frac{dx}{ds} = (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}} \quad \text{and} \quad \frac{dy}{ds} = (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}} \cdot \cosh \frac{x}{c}.$$

$$\therefore \frac{d^2x}{ds^2} = -\frac{1}{2} (1 + \cosh^2 \frac{x}{c})^{-\frac{3}{2}} \cdot 2 \cosh \frac{x}{c} \sinh \frac{x}{c} \cdot \frac{1}{c} \cdot (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}}$$

$$= -\frac{1}{c} \frac{\sinh(x/c) \cosh(x/c)}{[1 + \cosh^2(x/c)]^2}.$$

$$\begin{aligned} \frac{d^2 y}{ds^2} = & -\frac{1}{2} \left(1 + \cosh^2 \frac{x}{c}\right)^{-\frac{3}{2}} \cdot 2 \cosh \frac{x}{c} \sinh \frac{x}{c} \cdot \frac{1}{c} \cdot \left(1 + \cosh^2 \frac{x}{c}\right)^{-\frac{1}{2}} \cdot \cosh \frac{x}{c} \\ & + \sinh \frac{x}{c} \cdot \frac{1}{c} \left(1 + \cosh^2 \frac{x}{c}\right)^{-\frac{1}{2}} \cdot \left(1 + \cosh^2 \frac{x}{c}\right)^{-\frac{1}{2}} = \frac{1}{c} \frac{\sinh(x/c)}{[1 + \cosh^2(x/c)]^2}. \end{aligned}$$

Also solved by H. C. Feemster, G. B. M. Zerr, V. M. Spunar, and J. Scheffer.

278. Proposed by S. A. COREY, Hiteman, Iowa.

If C be Euler's constant, .577,215,664,9... and if B_1, B_2, B_3 , etc., be Bernoulli's numbers, $\frac{1}{6}, \frac{1}{36}, \frac{1}{42}$, etc., prove that

$$C = \frac{1}{2} + \frac{B_1}{2} - \frac{B_2}{4} + \frac{B_3}{6} - \frac{B_4}{8} + \dots - (1)^m \frac{B_m}{2m} + \dots$$

I. Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Euler's constant C may be presented under various forms from among which an elementary one may be the following:

$$C = \lim_{n \rightarrow \infty} \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots - \frac{1}{n} \log n \right] = .577,215,664,9\dots$$

By Taylor's Theorem, we have

$$f(a+h) - f(a) = hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \dots + \frac{h^n}{n!} f^{(n)}(a) + \dots$$

Change a successively into $a+h, a+2h, \dots, a+(n-1)h$ and add, then if we put x for $(a+nh)$, we obtain the following result:

$$f(x) - f(a) = h \Sigma f'(x) + \frac{h^2}{2!} \Sigma f''(x) + \frac{h^3}{3!} \Sigma f'''(x) + \dots + \frac{h^n}{n!} \Sigma f^{(n)}(x) + \dots$$

where $\Sigma f'(x) = f'(x) + f'(a+h) + \dots, \Sigma f''(x) = f''(a) + f''(a+h) + \dots$, etc.

Let $\phi(x) = f'(x)$. Then

$$\int_a^{a+nh} \phi(x) dx = h \Sigma \phi(x) + \frac{h^2}{2!} \Sigma \phi'(x) + \frac{h^3}{3!} \Sigma \phi''(x) + \dots + \frac{h^n}{n!} \Sigma \phi^{(n-1)}(x) + \dots$$